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Density of states and Fermi level of a periodically modulated two-dimensional electron gas

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Abstract

Explicit analytic expressions are obtained for the density of states $D(E)$ and Fermi energy E_F of a two-dimensional electron gas in the presence of a weak and periodic unidirectional electric or magnetic modulation and of a uniform perpendicular magnetic field B. The Landau levels broaden into bands and their width, proportional to the modulation strength, oscillates with B and gives rise to Weiss oscillations in $D(E)$, E_F and the transport coefficients. When both *electric* and *magnetic* modulations are present the position of the resulting oscillations depends on the ratio δ between the two modulation strengths. When the modulations are *out of phase* there is no shift in the position of the oscillations when δ varies and for a particular value of δ the oscillations are suppressed.

1. Introduction

The transport coefficients of a two-dimensional electron gas (2DEG) subjected to weak and periodic *electric* or *magnetic* modulations, along one or two directions, oscillate [1] as a function of the perpendicular magnetic field B. These so-called *Weiss* oscillations are now well established both theoretically [2] and experimentally [3, 4]. The modulation broadens the Landau levels into bands and their width, proportional to the strength of the modulation, oscillates with the strength of the field B. These oscillations reflect the commensurability between two length scales: the cyclotron diameter at the Fermi level $2R_c = 2\sqrt{2\pi n_e}\ell^2$ (where n_e is the electron density and $\ell = \sqrt{\hbar/eB}$ the magnetic length) and the period of the modulation a. For a *magnetic* modulation the phase of the oscillations is shifted by $\pi/2$ with respect to those occurring when an *electric* modulation is present.

The Weiss oscillations occur in weak fields B, prior to the Shubnikov–de Haas (SdH) oscillations, and are robust against temperature, i.e., they disappear at substantially higher temperatures than the SdH oscillations. They also show up in the thermodynamic quantities such as the Fermi level and the density of states (DOS). Despite numerous studies, to our knowledge these quantities have been evaluated only numerically. It is of interest to have as explicit and simple expressions for them as possible, valid for the relevant weak magnetic fields. This would be useful in magnetocapacitance experiments that directly probe the DOS at the Fermi level, especially for the less studied *magnetic* modulations.

The purpose of this paper is to provide analytic expressions for the DOS and the Fermi level valid in the presence of either modulation or both modulations. The reason for considering the latter case is that from the experimentally known methods of *magnetic* modulation one expects that the magnetic [5] or superconducting stripes [6], periodically placed on top of a 2DEG, act like electrical gates and induce an *electric* modulation of the 2DEG. As shown in our previous work on transport [7, 8], the relative phase of the two modulations can have important consequences and even suppress the *Weiss* oscillations as was confirmed experimentally [9].

The paper is organized as follows. In section 2 we briefly present the energy spectrum of a modulated 2DEG in the presence of a normal field B and point out some of its consequences. In sections 3 and 4 we evaluate, respectively, the broadened DOS and the Fermi level for either modulation alone. Then in section 5 we indicate how these results are modified when both modulations are present. Concluding remarks follow in the last section.

2. Energy spectrum

We consider a 2DEG, in the (x, y) plane, subjected to a perpendicular magnetic field $\boldsymbol{B}$ and to a *weak*, *electric*, $U(x)$, or *magnetic*, $B(x)$, periodic potential. In the first case we take $U(x) = V_0 \cos(Kx)$, $K = 2\pi/a$, where a is the modulation period, and use the Landau gauge for the vector potential $\boldsymbol{A} = (0, Bx, 0)$. In the effective-mass approximation the one-electron Hamiltonian is $H = (\boldsymbol{p} + e\boldsymbol{A})^2/2m^* + U(x)$, where $p_\mu (\mu = x, y)$ is the momentum operator. In the second case we consider the vector potential $\boldsymbol{A} = (0, Bx + (B_0/K)\sin(Kx), 0)$ that describes a periodically modulated magnetic field $\boldsymbol{B} = (B + B_0 \cos(Kx))\hat{z}$ and take $B_0 \ll B$; this implies $\omega_0 = eB_0/m^* \ll \omega_c = eB/m^*$, where ω_c is the cyclotron frequency. The one-electron Hamiltonian is simply $H = (\boldsymbol{p} + e\boldsymbol{A})^2/2m^*$ and in the absence of the modulation, i.e., for $U(x) = 0$ or $\omega_0 = 0$, the normalized eigenvector is $\psi_{nk_y} = \mathrm{e}^{\mathrm{i}k_y y}\phi_n(x - x_0)/\sqrt{L_y}$, where $x_0 = -\ell^2 k_y = -\hbar k_y/eB$ is the centre coordinate of the cyclotron orbit, L_y the width of the sample in the y direction and n the Landau level index. $\phi_n(x)$ is the well known harmonic oscillator wavefunction. In the absence of modulation the eigenvalues are $E_{nk_y} = (n+1/2)\hbar\omega_c$ which are degenerate with respect to the wavevector k_y along the y-direction.

In the presence of a weak modulation the energy correction to E_{nk_y} is obtained [7] by first-order perturbation theory using the unperturbed eigenstates ψ_{nk_y}. To first order in the modulation strength the energy spectrum is

$$E_{nk_y} = (n + 1/2)\hbar\omega_c + V_0 F_n(u)\cos(Kx_0) \tag{1}$$

for an *electric* modulation, and

$$E_{nk_y} = (n + 1/2)\hbar\omega_c + \hbar\omega_0 G_n(u)\cos(Kx_0) \tag{2}$$

for a *magnetic* modulation. Here $u = K^2\ell^2/2$, $F_n(u) = \mathrm{e}^{-u/2}L_n(u)$ and $G_n(u) = \mathrm{e}^{-u/2}[L_n(u)/2 + L^1_{n-1}(u)] = -\partial F_n(u)/\partial u$, $L_n(u)$ is a Laguerre polynomial. Since $x_0 = -\ell^2 k_y$, the k_y degeneracy of the energy levels is lifted and the energy levels broaden into bands. This energy spectrum can also be derived semiclassically as shown in [7].

For the experimentally relevant *weak* magnetic fields B it is a good approximation to take the large-n limit of the Laguerre polynomials that appear in $G_n(u)$ and $F_n(u)$. For the *electric* modulation the width of the Landau level at the Fermi energy is then obtained as

$$2V_0|F_n(u)| \approx 2V_0(2/\pi K R_c)^{1/2}|\cos(KR_c - \pi/4)|. \tag{3}$$

The corresponding result for the *magnetic* modulation is obtained by replacing V_0 by $\hbar\omega_0$ and $F_n(u)$ by $G_n(u)$; it reads

$$2\hbar\omega_0|G_n(u)| \approx 2\hbar\omega_0(ak_F/2\pi)(2/\pi K R_c)^{1/2}|\sin(K R_c - \pi/4)|. \tag{4}$$

Here $R_c = (2n_F + 1)^{1/2}\ell$ is the cyclotron radius at the Fermi energy and n_F is defined as the largest integer contained in $E_F/\hbar\omega_c - 1/2$. From equation (4) we obtain the flat-band condition as $2R_c/a = i + 1/4$; the maximum bandwidth occurs for $2R_c/a = i + 3/4$, with $i = 0, 1, 2, \ldots$.

Equations (3) and (4) are valid in the low-magnetic-field limit, typically $B \leqslant 1$ T, in which case we have $n_F \gg 1$ for typical two-dimensional electron densities. Comparing equation (3) with (4) we see that (1) there is a $\pi/2$ *phase shift* between the bandwidths, and (2) the amplitude in the *magnetic* modulation case is larger by a factor $ak_F/2\pi = \sqrt{n_e a^2/2\pi}$ (=7.6 for $n_e = 3 \times 10^{11}$ cm^{-2} and $a = 3500$ Å) than that of the *electric* case for equal modulation strengths. An illustration of that is given in [7] in which the effect of the modulation on the resistivity is evaluated.

3. Density of states

The qualitative differences in the energy spectrum, with or without either modulation, are also reflected in the DOS: $D(E) = \sum_{n,k_y} \delta(E - E_{n,k_y})$, expressed per unit area surface. With $t = K x_0$ the DOS takes the form

$$\frac{D(E)}{D_0} = \frac{\hbar\omega_c}{2} \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}t\, \delta(E - E_{n,t}), \tag{5}$$

where $D_0 = m^*/\pi\hbar^2$ is the DOS of a free 2DEG at $B = 0$. When evaluated numerically [8] the DOS shows van Hove singularities at the edges of the modulation-broadened Landau levels. This reflects the one-dimensional nature of the electron motion.

In practical systems this DOS will always be broadened due to the presence of scattering centres. This is reasonable because we limit ourselves to the case of the experimentally important weak magnetic fields. The results for a Gaussian broadening are given in the appendix. Here we assume a Lorentzian broadening of zero shift and constant width Γ; in this case equation (5) takes the form

$$\frac{D(E)}{D_0} = \frac{\hbar\omega_c}{2} \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}t \frac{\Gamma/\pi}{(E - E_{n,t})^2 + \Gamma^2}. \tag{6}$$

Here and in equation (5) we have $E_{n,t} = E_n + \Delta_n \cos t$ with $\Delta_n = V_0 F_n(u)$ or $\Delta_n = \hbar\omega_0 G_n(u)$. In order to get the correct low-energy limit of $D(E)$ we have to multiply the right-hand side of equation (6) by $N(E_{nt}) = [1/2 + \arctan(E_{nt}/\Gamma)/\pi]^{-1}$, which is the negative energy contribution of the broadened Landau levels to the DOS.

For weak modulation strengths we expand the integrand of equation (6) in powers of $\epsilon_{n,t} = \Delta_n \cos t$. The odd powers of $\epsilon_{n,t} = \Delta_n \cos t$ vanish after integration over t. To second order in the modulation potential the even powers are easily integrated. Taking spin into account by multiplying the result by 2 the DOS becomes

$$\frac{D(E)}{D_0} = \frac{\hbar\omega_c}{\pi} \Gamma \sum_{n=0}^{\infty} \frac{1}{(E - E_n)^2 + \Gamma^2} \left\{ 1 + \frac{\Delta_n^2}{2} \frac{3(E - E_n)^2 - \Gamma^2}{[(E - E_n)^2 + \Gamma^2]^2} \right\}. \tag{7}$$

For $E \approx E_n$, i.e., near the centre of the nth Landau level, the expression is valid only when $|\Delta_n| \leqslant \Gamma$. The sum over n is carried out using Poisson's summation formula

$$\sum_{n=0}^{\infty} f(n + 1/2) = \int_0^{\infty} f(x)\, \mathrm{d}x + 2 \sum_{n=1}^{\infty} (-1)^n \int_0^{\infty} f(x) \cos(2n\pi x)\, \mathrm{d}x. \tag{8}$$

3.1. Electric modulation

To evaluate the first term in the square brackets of equation (7) we proceed as follows. Since $E \sim E_F \gg \Gamma$ we can replace $-E/\Gamma$ by $-\infty$. Then the corresponding first integral on the right-hand side, with $n + 1/2 \to x$, gives 1. With $y = (x\hbar\omega_c - E)/\Gamma$ the second integral becomes proportional to $\int_{-\infty}^{\infty} \mathrm{d}y(1 + y^2)^{-1} \cos(2n\pi\Gamma y/\hbar\omega_c)$ which is equal to $\pi \exp(-2n\pi\Gamma/\hbar\omega_c)$. The final result for this term, labelled D_1, is

$$D_1 = 1 + 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-2n\pi\bar{\Gamma}} \cos(2n\pi\bar{E}), \tag{9}$$

where $\bar{\Gamma} = \Gamma/\hbar\omega_c$ and $\bar{E} = E/\hbar\omega_c$. D_1 gives the Lorentzian-broadened DOS of an unmodulated 2DEG. The limit $\bar{\Gamma} \to 0$ gives the corresponding unbroadened DOS, i.e., a series of delta functions at the Landau levels.

We denote the second term in the square brackets of equation (7) by D_2. We now use the asymptotic expression of $\Delta_n = V_0 F_n(u)$ given by equation (3), the replacement of n by $n + 1/2$ since we have $n \gg 1$, the expansion $(1 + z)^{1/2} \approx 1 + z/2$ for $|z| = \Gamma|y|/E \ll 1$ because $E \approx E_F$ and the previous approximations used above. The result corresponding to the first integral is

$$-\frac{1}{4\pi}\left(\frac{\bar{V}_0}{\bar{E}}\right)^2 \alpha \mathrm{e}^{-\alpha\bar{\Gamma}/\bar{E}}[\sin(2\alpha) + \cos(2\alpha)/\alpha], \tag{10}$$

where $\alpha = \alpha(E) = \sqrt{2\bar{E}}K\ell$ and $\bar{V}_0 = V_0/\hbar\omega_c$. With the help of the formula $\int_0^{\infty} \mathrm{d}x \cos(ax)/(b^2 + x^2) = (\pi/2|b|)\exp(-|ab|)$ the second integral of equation (8) can be carried out but the result is unwieldy. With the approximation $2n\pi\bar{E} \approx 2n\pi E_F/\hbar\omega_c \gg 1$ we obtain

$$-\frac{4\pi}{\alpha}\bar{V}_0^{\,2} \cos^2(\alpha - \pi/4) \sum_{n=1}^{\infty}(-1)^n n^2 \mathrm{e}^{-2n\pi\bar{\Gamma}} \cos(2n\pi\bar{E}). \tag{11}$$

Collecting equations (9)–(11), the end result for the DOS is

$$\frac{D(E)}{D_0} = 1 - \frac{1}{4\pi}\left(\frac{\bar{V}_0}{\bar{E}}\right)^2 \alpha \mathrm{e}^{-\alpha\bar{\Gamma}/\bar{E}}[\sin(2\alpha) + \cos(2\alpha)/\alpha]$$
$$+ 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-2n\pi\bar{\Gamma}} \cos(2n\pi\bar{E})\left[1 - \frac{2\pi n^2}{\alpha}\bar{V}_0^{\,2} \cos^2(\alpha - \pi/4)\right]. \tag{12}$$

The modulation introduces two correction terms that are of order V_0^2, one in the constant background term and one in the SdH oscillation term.

Figure 1(a) shows the DOS as a function of energy. The solid curve shows the result from the analytic expression (12) and the dashed one is obtained numerically from equation (6). The parameters used in the numerical calculation are $V_0 = 0.5$ meV, $a = 350$ nm, $\Gamma = 4$ K and $B = 0.58$ T. As can be seen, the agreement between the two curves is good for most of the energies. Discrepancies are found near the maxima of the DOS, i.e., at the position of the Landau levels, where $|\Delta_n| > \Gamma$ and the expansion (7) is invalid. In figure 1(b) we show the DOS at the Fermi energy as a function of the magnetic field. The Fermi energy is fixed at $E_F = 10$ meV. Good agreement is found except at certain magnetic fields for which the amplitude of the SdH oscillations is slightly underestimated.

3.2. Magnetic modulation

The difference from the previous case is that now $\Delta_n = \hbar\omega_0 G_n(u) \approx \hbar\omega_0(n/u)^{1/4}(\pi u)^{-1/2} \sin(2\sqrt{nu} - \pi/4)$. The first term in equation (7), now labelled M_1,

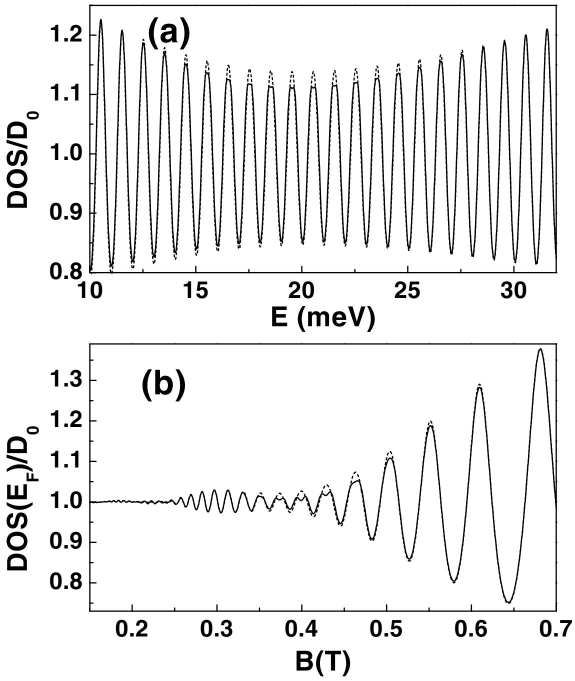

Figure 1. (a) The DOS of a 2DEG in a periodic electric modulation as a function of energy. (b) The DOS at the Fermi energy as a function of the magnetic field. The solid curves result from equation (12) and the dashed ones are from the integration in equation (6).

is independent of the modulation and is given again by equation (9). We label the second term M_2 and use the same approximations as above. The result for the first integral on the right-hand side of equation (8), labelled M_{21}, is

$$M_{21} = \frac{1}{\pi\alpha}\left(\frac{\omega_0}{\omega_c}\right)^2 \mathrm{e}^{-\alpha\bar{\Gamma}/\bar{E}}[\sin(2\alpha) - \cos(2\alpha)/\alpha]. \tag{13}$$

If we make the same approximations as in the electric modulation case, the result for the second integral on the right-hand side of equation (8), labelled M_{22}, is

$$M_{22} = -\frac{4\pi\alpha}{(K\ell)^4}\left(\frac{\omega_0}{\omega_c}\right)^2 \sin^2(\alpha - \pi/4)\sum_{n=1}^{\infty}(-1)^n n^2 \mathrm{e}^{-2n\pi\bar{\Gamma}}\cos(2n\pi\bar{E}). \tag{14}$$

Combining equations (9), (13) and (14) we obtain

$$\frac{D(E)}{D_0} = 1 + \frac{1}{\pi\alpha}\left(\frac{\omega_0}{\omega_c}\right)^2 \mathrm{e}^{-\alpha\bar{\Gamma}/\bar{E}}[\sin(2\alpha) - \cos(2\alpha)/\alpha]$$
$$+ 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-2n\pi\bar{\Gamma}}\cos(2n\pi\bar{E})\left[1 - \frac{2\pi\alpha n^2}{(K\ell)^4}\left(\frac{\omega_0}{\omega_c}\right)^2 \sin^2(\alpha - \pi/4)\right]. \tag{15}$$

Figure 2(a) shows the DOS as a function of energy. The solid curve is the approximate result from equation (15) and the dashed one the numerical result from equation (6). The parameters in the calculation are $B_0 = 0.04$ T, $B = 0.58$ T, $a = 350$ nm and $\Gamma = 4$ K. In figure 2(b) we show the DOS at the Fermi energy as a function of the magnetic field. The Fermi energy is fixed at $E_F = 10$ meV. The overall agreement is rather good. The disagreement around

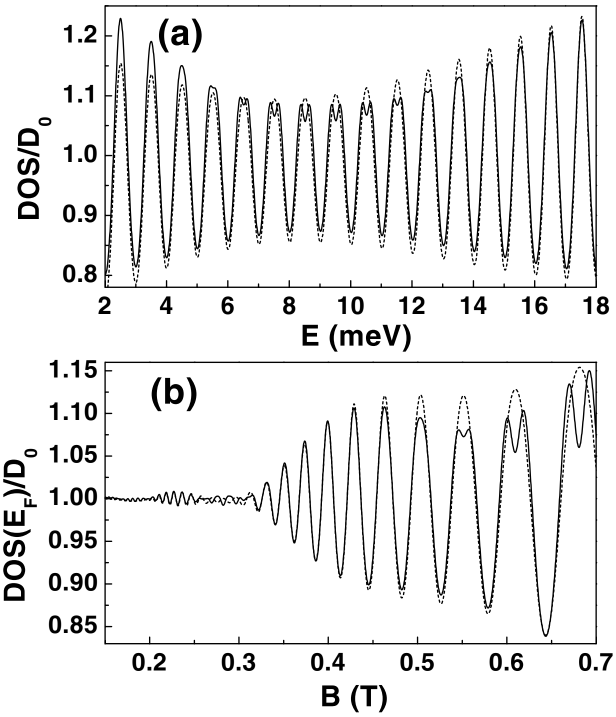

Figure 2. (a) The DOS of a 2DEG in a periodic magnetic modulation as a function of energy. (b) The DOS at the Fermi energy as a function of the magnetic field. The solid curves result from equation (15) and the dashed ones from equation (6).

$B = 0.28$ T and the dips at the DOS maxima for $B > 0.5$ T are due to the fact that the condition $|\Delta_n| \leqslant \Gamma$, used in obtaining equation (7), is not satisfied.

4. Fermi level

4.1. Electric modulation

With a constant electron density n_e the Fermi level is determined by

$$n_e \pi \ell^2 = \sum_{n=0}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} f(E_{n,t})\, \mathrm{d}t, \tag{16}$$

where $f(E) = 1/(\mathrm{e}^{(E-E_F)/k_B T} + 1)$ is the Fermi–Dirac distribution function, $E_{n,t} = E_n + \Delta_n \cos t$ and $t = K x_0$. If we expand the Fermi–Dirac function $f(E_{n,t})$ in powers of $\Delta_n \cos t$ and integrate over t the odd powers of $\cos t$ vanish and equation (16), to second order in the modulation strength, takes the form

$$n_e \pi \ell^2 = \sum_{n=0}^{\infty} \frac{1}{\pi} \int_0^{\pi} \left[f(E_n) + \frac{\Delta_n^2}{2} f''(E_n) \cos^2 t \right] \mathrm{d}t. \tag{17}$$

Integrated over t the first term on the right-hand side gives π and the second one, proportional to $\cos^2 t$, gives $\pi/2$. Next we use the asymptotic expression for Δ_n, insert the explicit expression

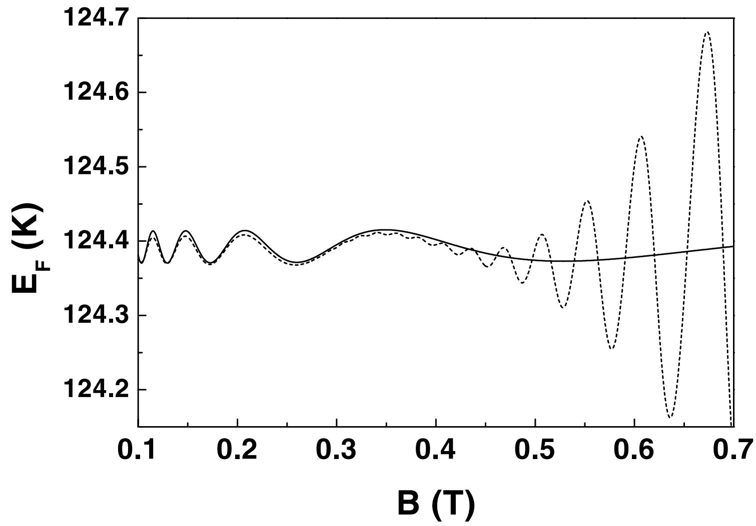

Figure 3. Fermi energy versus magnetic field for the electric modulation case. The solid curve results from equation (20) and the dashed one from equation (16).

for $f''(E_n)$, and replace the sum over $n \approx E/\hbar\omega_c$ by an integral over E; this results in

$$n_e\pi\ell^2 = \int_0^\infty \frac{\mathrm{d}E}{\hbar\omega_c}\left\{f(E) + \frac{V_0^2\beta^2}{8\pi\alpha}\frac{\sinh[\beta(E-E_F)/2]}{\cosh^3[\beta(E-E_F)/2]}\cos^2(\alpha-\pi/4)\right\}, \qquad (18)$$

where $\beta = 1/k_BT$. The first term gives $E_F/\hbar\omega_c$. The second term is nonzero near the Fermi energy, $|E-E_F| \leqslant k_BT \ll E_F$. With $E = E_F(1+x)$, $|x| \ll 1$, making the approximation $(1+x)^{1/2} \approx 1+x/2$ in the argument of $\cos^2[\cdots]$, and $(1+x)^{-1/2} \approx 1-x/2$ in the denominator, we get

$$(1-x/2)\cos^2[\alpha_F(1+x/2)-\pi/4], \qquad (19)$$

where $\alpha_F = \alpha(E_F) = KR_c$. Expanding the $\cos^2[\cdots]$ term in powers of x and inserting the result in equation (18) we see immediately that the even powers of x give, upon integration, zero. The first nonvanishing result comes from the term $\propto x$. To second order in V_0 the final result for E_F, including both terms, is obtained:

$$E_F = E_F^0 - \frac{V_0^2}{4\pi E_F}\left[\cos(2KR_c) - \frac{1}{KR_c}\cos^2(KR_c-\pi/4)\right], \qquad (20)$$

where $E_F^0 = n_e\pi\hbar^2/m^*$ is the Fermi energy of a free 2DEG in zero magnetic field. E_F can be obtained by solving equation (20). In figure 3 we show the Fermi energy versus magnetic field as it results from equation (20) (solid curve) and from equation (16) (dashed curve). The parameters in the calculation are $V_0 = 0.5$ meV, $a = 350$ nm, $\Gamma = 4$ K, $T = 1$ K and $n_e = 3 \times 10^{11}$ cm^{-2}. As can be seen, for weak magnetic fields ($B < 0.5$ T) the agreement between the two curves is rather good. Clear differences are found on the right half where the result from equation (16) shows the SdH oscillations while the semiclassical one misses them, as expected. The phase and period of the Weiss oscillations in both results are in perfect agreement.

4.2. Magnetic modulation

We follow verbatim the procedure just outlined for the electric modulation. Using the corresponding asymptotic expression for Δ_n in equation (17) we find again equation (18)

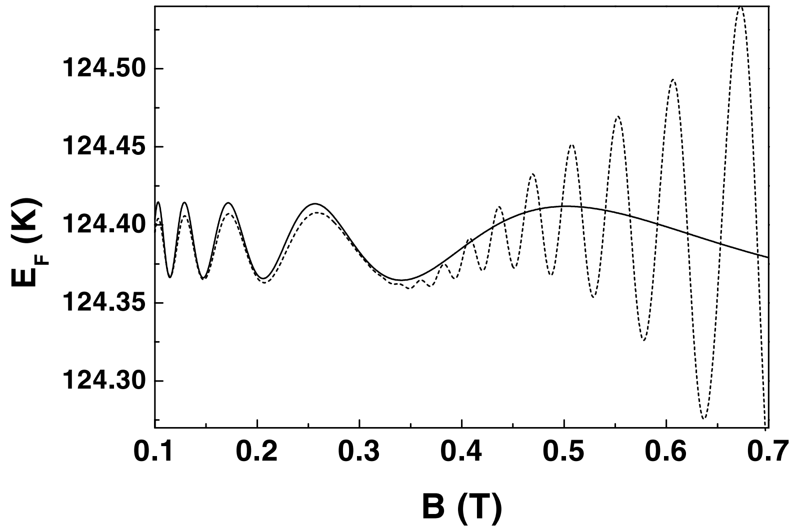

Figure 4. Fermi energy versus magnetic field for the magnetic modulation case. The solid curve results from equation (21) and the dashed one from equation (16).

with V_0 replaced by $\hbar\omega_0$, $\cos^2(\alpha - \pi/4)$ by $\sin^2(\alpha - \pi/4)$ and $1/\alpha$ by $\alpha/(K\ell)^4$. The integrals are performed as above and the end result for E_F is given by

$$E_F = E_F^0 + \frac{1}{2\pi(K\ell)^2}\frac{(\hbar\omega_0)^2}{\hbar\omega_c}\left[\cos(2KR_c) - \frac{1}{KR_c}\sin^2(KR_c - \pi/4)\right]. \quad (21)$$

In figure 4 we show the Fermi energy versus magnetic field as it results from equation (21) (solid curve) and from equation (16) (dashed curve). The parameters in the calculation are $B_0 = 0.04$ T, $a = 350$ nm, $\Gamma = 4$ K, $T = 1$ K and $n_e = 3 \times 10^{11}$ cm^{-2}. As can be seen, the overall agreement between the two curves is rather good. The differences are the same as for the electric modulation but now the SdH oscillations are seen more clearly in the integration result from equation (16).

5. Electric and magnetic modulations

Both methods of *magnetic* modulation mentioned in section 1 are expected to be accompanied by an *electric* one since gates are applied to the heterostructure. We are thus led to study how the results of section 3 are modified when an *electric* and a *magnetic* modulation with the same period are present. Two cases are of interest: when the two modulations are *in phase* and when they are *out of phase*.

5.1. In-phase modulations

When both modulations treated in section 2 are present and are *in phase*, the energy spectrum, to first order in the modulation strengths, reads

$$E_{nk_y} = (n + 1/2)\hbar\omega_c + [V_0F_n(u) + \hbar\omega_0G_n(u)]\cos(Kx_0). \quad (22)$$

At the Fermi energy the bandwidth is equal to

$$2V_0\sqrt{2/\pi KR_c}\sqrt{1+\delta_F^{-2}}\sin(KR_c - \pi/4 + \phi), \quad (23)$$

where $\delta_F = KV_0/k_F\hbar\omega_0 = \tan\phi$. Notice that the flat-band condition now reads $2R_c/a = i + 1/4 - \phi/\pi$ and depends on the relative strength of the two modulations.

The changes mentioned above will be reflected in the DOS and the Fermi level as well as in the transport coefficients treated before. The DOS will have the terms appearing in equations (12) and (15) and extra terms resulting from the cross term in Δ_n^2, namely $2V_0F_n(u)\hbar\omega_0G_n(u)$. The asymptotic expression for this cross term is equal to $(V_0\hbar\omega_0/\pi u)\sin(4\sqrt{nu} - \pi/2)$. Using the same procedure as above the DOS is obtained as

$$\frac{D(E)}{D_0} = 1 - \frac{1}{4\pi}\left(\frac{\bar{V}_0}{\bar{E}}\right)^2 \alpha e^{-\alpha\bar{\Gamma}/\bar{E}}[(1-\delta^{-2})\sin(2\alpha) - 2\cos(2\alpha)/\delta + (1+\delta^{-2})\cos(2\alpha)/\alpha] + 2\sum_{n=1}^{\infty}(-1)^n e^{-2n\pi\bar{\Gamma}}\cos(2n\pi\bar{E})\left\{1 - \frac{\pi n^2}{\alpha}\bar{V}_0^{\,2}[(1-\delta^{-2})\sin(2\alpha) - 2\cos(2\alpha)/\delta + (1+\delta^{-2})]\right\}, \tag{24}$$

where $\delta = \delta(E) = (V_0/\hbar\omega_0)(K^2\ell^2/\alpha)$. In figure 5(a) we show the DOS as a function of energy with in-phase electric and magnetic modulations. The solid curve is the result from the analytical expression in equation (24) and the dashed one the numerical result obtained from equation (6). The parameters in the calculation are $a = 350$ nm, $B_0 = 0.04$ T, $V_0 = 0.2$ meV, $B = 0.58$ T and $\Gamma = 4$ K. The dips at the Landau levels for $E < 10$ meV are due to the fact that the condition $|\Delta_n| < \Gamma$ is not satisfied. Figure 5(b) shows the DOS at the Fermi energy as a function of the magnetic field. The Fermi energy is fixed at $E_F = 10$ meV. The disagreement around $B = 0.3$ T and the dips at the DOS maxima for $B > 0.6$ T are due to the same factor, i.e., the condition $|\Delta_n| < \Gamma$ is not satisfied.

The result for the Fermi level will have the terms of equations (20) and (21) and an extra term resulting from the same cross term in Δ_n^2. Using the same procedure as above the contribution of this term is obtained as $-(V_0\hbar\omega_0/\pi KR_c\hbar\omega_c)\sin(2KR_c)$. In terms of δ_F and V_0 the full result for the Fermi level can be written as

$$E_F = E_F^0 + \frac{V_0^2}{4\pi E_F}\{(1-\delta_F^{-2})[(2KR_c)^{-1} - \cos(2KR_c)] + [(1+\delta_F^{-2})(2KR_c)^{-1} - 2\delta_F^{-1}]\sin(2KR_c)\}. \tag{25}$$

It is interesting to notice that the flat-band condition $2R_c/a = i + 1/4 - \phi/\pi$ together with $\delta_F = \delta(E_F) = (V_0/\hbar\omega_0)(K/k_F) = 1 = \tan\phi$ simplify the result for E_F considerably: the last term in equation (25), proportional to $\sin(2KR_c)$ vanishes and the second term $\sim V_0^2$ on the first line can be neglected.

Figure 6 shows the Fermi energy versus magnetic field as it results from equation (25) (solid curve) and from equation (16) (dashed curve). The parameters used in the calculation are $a = 350$ nm, $B_0 = 0.04$ T, $V_0 = 0.2$ meV, $\Gamma = 4$ K, $T = 1$ K and $n_e = 3\times10^{11}$ cm^{-2}. We see again that the Weiss oscillations on the two curves agree well.

5.2. *Out-of-phase modulations*

If light is shone on top of a magnetically modulated heterostructure the light pulses will ionize DX centres in the AlGaAs layer between the gates (e.g. stripes of magnetic materials) where $B_0 = 0$. This will create an *electric* modulation $\pi/2$ *out of phase* with the *magnetic* one. Assuming that the *electric* modulation of the gates is much smaller than that of the light, something that can be achieved by contacting the gates, we are led to consider transport in the presence of the two modulations that are $\pi/2$ *out of phase*.

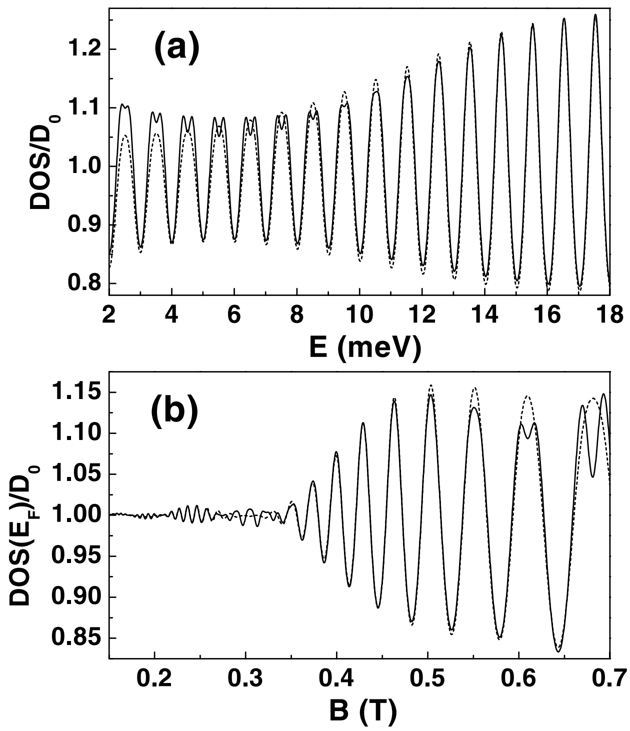

Figure 5. (a) The DOS of a 2DEG as a function of energy for in-phase electric and magnetic modulations. (b) The DOS at the Fermi energy as a function of the magnetic field. The solid curves result from equation (24) and the dashed ones are results from equation (6).

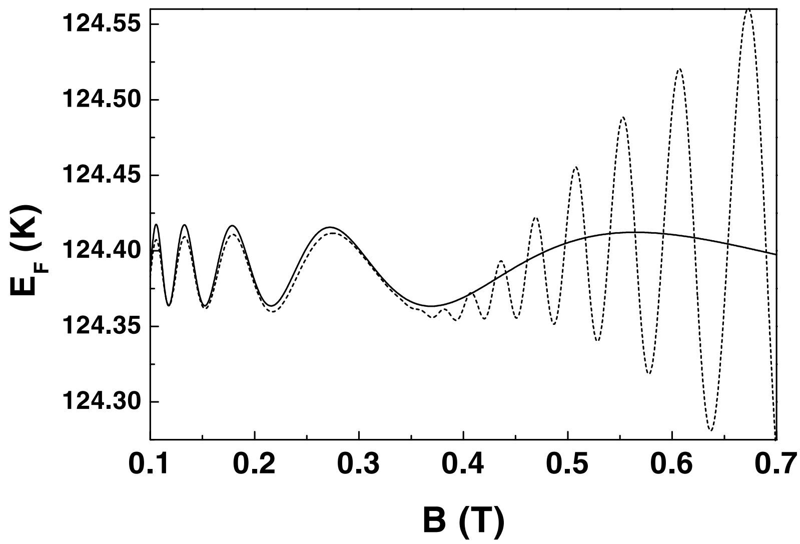

Figure 6. Fermi energy versus magnetic field for in-phase electric and magnetic modulations. The solid curve results from equation (25) and the dashed one from equation (16).

We consider the same *electric* modulation $V_0\cos(Kx)$ and assume a *magnetic* one described by $B(x) = B + B_0\sin(Kx)$. To first order in V_0 and B_0 the eigenvalue E_{nk_y} is

$$E_{nk_y} = (n + 1/2)\hbar\omega_c + \hbar\omega_0 G_n(u)\sin(Kx_0) + V_0 F_n(u)\cos(Kx_0). \tag{26}$$

This change and the corresponding one for the velocity have important consequences for the transport coefficients [7] and for other quantities. For instance, the combined bandwidth at the Fermi energy now is equal to

$$2V_0\sqrt{2/\pi KR_c}\sqrt{1 + (\delta_F^{-2} - 1)\sin^2(KR_c - \pi/4)}. \tag{27}$$

If $\delta_F = \pm 1$ the bandwidth no longer oscillates as a function of the magnetic field, i.e., the Weiss oscillations are washed out.

There is no cross term involving the product $\hbar\omega_0 V_0$ in the expression for the DOS or the Fermi level because in either of them the relevant integrand over t, once the expansion in powers of Δ_n is made, is $\cos t \sin t$ which upon integration gives a zero contribution. Hence, in this case the DOS is given by

$$\begin{aligned}\frac{D(E)}{D_0} = 1 &- \frac{1}{4\pi}\left(\frac{\bar{V}_0}{\bar{E}}\right)^2 \alpha \mathrm{e}^{-\alpha\bar{\Gamma}/\bar{E}}[(1 - \delta^{-2})\sin(2\alpha) + (1 + \delta^{-2})\cos(2\alpha)/\alpha] \\ &+ 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-2n\pi\bar{\Gamma}}\cos(2n\pi\bar{E}) \\ &\times \left\{1 - \frac{\pi n^2}{\alpha}\bar{V}_0^{\,2}[(1 + \delta^{-2}) + (1 - \delta^{-2})\sin(2\alpha)]\right\}.\end{aligned} \tag{28}$$

Figure 7(a) shows the DOS as a function of energy. The solid curve results from equation (28) and the dashed one from equation (6). The parameters in the numerical calculation are taken the same as in figure 5. Figure 7(b) shows the DOS at the Fermi energy as a function of the magnetic field.

The Fermi level is now obtained as

$$E_F = E_F^0 + \frac{V_0^2}{4\pi E_F}\{(1 - \delta_F^{-2})[2KR_c)^{-1} - \cos(2KR_c)] + (1 + \delta_F^{-2})(2KR_c)^{-1}\sin(2KR_c)\}. \tag{29}$$

As can be seen, the condition $\delta_F = 1$ is now sufficient for the Fermi level to be given approximately by the zero-field term E_F^0 since all other terms, of order $1/\alpha_F = 1/KR_c \ll 1$, can be neglected. Accordingly, the Weiss oscillations are suppressed for $\delta_F = 1$. This suppression is in line with that found to occur in the transport coefficients [7]. Figure 8 shows the Fermi energy versus magnetic field as it results from equation (29) (solid curve) and from equation (16) (dashed curve).

6. Conclusions

We obtained explicit analytic expressions for the Weiss oscillations in the DOS and in the Fermi level E_F of a 2DEG in the presence of (i) an electric and (ii) a magnetic modulation. The oscillations in case (ii) are different from those in case (i) in two important aspects:

(1) there is a $\pi/2$ phase shift between the oscillations, and
(2) for equal modulation strengths, i.e., for $V_0 = \hbar\omega_c$, the oscillation amplitude in case (ii) is much larger than in case (i).

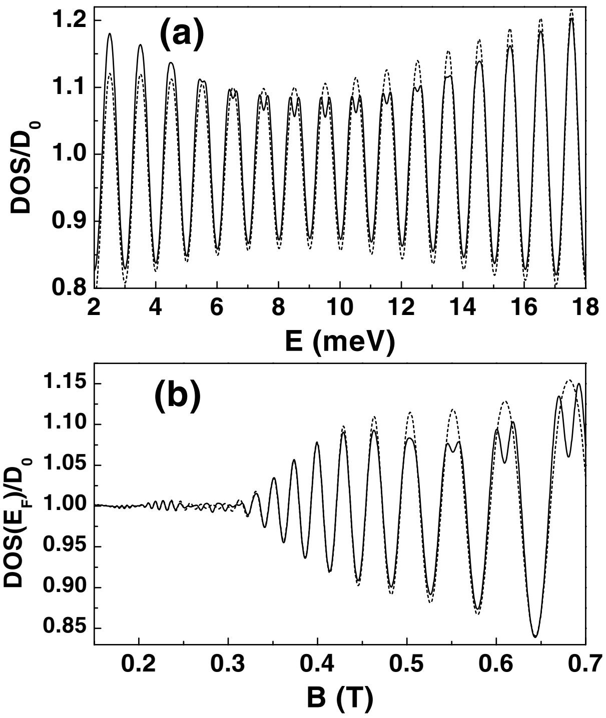

Figure 7. (a) The DOS of a 2DEG as a function of energy for out-of-phase electric and magnetic modulations. (b) The DOS at the Fermi energy as a function of the magnetic field. The solid curves are the results as given by equation (28) and the dashed ones are the results of equation (6).

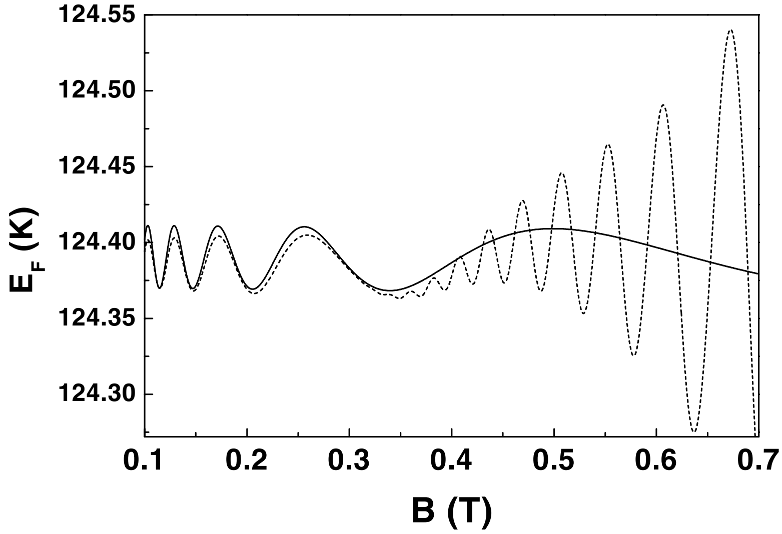

Figure 8. Fermi energy versus magnetic field for out-of-phase modulations. The solid curve results from the analytic expression given by equation (29) and the dashed one from equation (16).

In a real experimental system we expect that an electric modulation will inherently be present with each magnetic modulation. Therefore, we have also studied the case in which both types of modulation are present in a 2DEG. We found that if the modulations are *in phase* the extremal positions of the Weiss oscillations are shifted continuously with increasing strength of the electric oscillation. On the other hand, when the modulations are $\pi/2$ out of phase a different behaviour is found: with increasing electric modulation strength the position of the Weiss oscillations is not influenced but their amplitude is. If B_0 is kept constant for a critical value of the strength of the electric modulation the Weiss oscillations disappear and with a further increase of the modulation strength the Weiss oscillations reappear but now the maxima appear at the position of the previous minima.

Finally, as the figures demonstrate, there is an overall good agreement between the exact and the approximate results for the DOS and the Fermi energy. Discrepancies in the DOS could be found at the position of the Landau levels as the Landau band width is larger than the impurity broadening. As expected, the agreement in the Fermi energy holds only for the Weiss oscillations and not the SdH ones. Accordingly, in the regime of the former oscillations one can safely use the approximate results of the paper instead of the more involved exact ones.

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Appendix. Gaussian-broadened DOS

We replace the δ-function in equation (5) by a Gaussian of width Γ and expand the integrand in powers of $\Delta_n \cos t$. Corresponding to equation (7) we now obtain

$$\frac{D(E)}{D_0} = \frac{\hbar\omega_c}{\sqrt{\pi}}\frac{1}{\Gamma}\sum_{n=0}^{\infty} \mathrm{e}^{-(E-E_n)^2/\Gamma^2}\left[1 + \frac{\Delta_n^2}{2}\frac{2(E-E_n)^2-\Gamma^2}{\Gamma^4}\right]. \tag{.1}$$

The sum over n can be evaluated by applying the recipe (8) to equation (30). We make the same approximations as in the case of a Lorentzian broadening and use the formula $\int_0^\infty \exp(-ax^2)\cos(bx)\,\mathrm{d}x = (\pi/4a)^{1/2}\exp(-b^2/4a)$. After summing over n the first term in the square brackets of equation (30) gives equation (9) with $\exp(-2n\pi\bar{\Gamma})$ replaced by $\exp(-(n\pi\bar{\Gamma})^2)$. That is,

$$D_1 = 1 + 2\sum_{n=1}^{\infty}(-1)^n \mathrm{e}^{-(n\pi\bar{\Gamma})^2}\cos(2n\pi\bar{E}). \tag{.2}$$

Using similar approximations as before the corresponding result for the second term, labelled D_2, reads

$$D_2 \approx -\frac{\alpha}{4\pi}\left(\frac{\bar{V}_0}{\bar{E}}\right)^2 \mathrm{e}^{-(\alpha\bar{\Gamma}/2\bar{E})^2}[\sin(2\alpha) + \alpha^{-1}\cos(2\alpha)] \\ - \frac{4\pi}{\alpha}\bar{V}_0^{\,2}\cos^2(\alpha - \pi/4)\sum_{n=1}^{\infty}(-1)^n n^2 \cos(2n\pi\bar{E})\mathrm{e}^{-(n\pi\bar{\Gamma})^2}. \tag{.3}$$

Due to the factor $\exp(-(n\pi\Gamma/\hbar\omega_c)^2)$ in the summands the sums can be well approximated by the $n = 1$ term alone.

In a similar manner one can obtain the results for a magnetic modulation.

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